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A COOLING PROBLEM OF PEBBLE-BED NUCLEAR REACTORS

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Abstract—The fundamental equations for the steady-state flow of a gas through a porous medium with prescribed heat source are derived. Application of these equations enable the pressure drop and the flow distribution of a gaseous coolant through a pebble-bed type nuclear reactor to be calculated. One exact solution is given. The maximum gas temperature is found to be very much higher than the mixed mean value.

Résumé—Les équations fondamentales de l'écoulement permanent d'un gaz dans un milieux poreux ont été établies en présence d'une source de chaleur. L'application de ces équations conduit au calcul de la perte de charge et de la distribution de l'écoulement d'un fluide refroidisseur gazeux dans un réacteur du type à lit granuleux. Une solution exacte est obtenue. La température maximum du gaz est trouvée bien supérieure à celle donnée par la valeur moyenne du mélange.

Zusammenfassung—Für die stationäre Strömung eines Gases durch ein poröses Medium mit vorgegebener Wärmequelle werden die Grundgleichungen abgeleitet. Diese Gleichungen gestatten die Berechnung des Druckabfalls und der Strömungsverteilung eines Kühlgases beim Durchtritt durch einen Kernreaktor vom pebble-bed-Typ. Eine exakte Lösung wird mitgeteilt. Man findet, dass die höchste Gastemperatur sehr viel höher als der Mittelwert ist.

Abstract—Выведены основные уравнения переноса газа через пористую среду для стационарного режима с заданным тепловым источником. Применение этих уравнений позволяет вычислить перепад давлений и распределение скорости потока газа в среде крупнозернистого ядерного топлива реактора. Получено точное решение поставленной задачи. Найдено, что максимальная температура газа значительно превышает среднюю температуру смеси.

NOTATION

Any coherent set of units may be used.

- a, b, defined in equation (6);
- f defined in equation (18);
- F, F', defined in equations (4), (5);
- h, specific enthalpy;
- k, isentropic exponent;
- L, reactor core length;
- p, pressure;
- Q, heat generation per unit time and volume;
- r, radial co-ordinate;
- R, reactor core radius;
- s, root of equation (12);
- T, temperature;
- v, specific volume;
- w, $= r/R$;
- z, axial co-ordinate.
- Z, defined in equation (27)

Greek symbols

- β, defined in equation (18);
- θ, defined in equation (15);
- κ, defined in equation (21);
- λ $= k/(k-1)$;
- μ, defined in equation (15);
- ξ $= \pi z/L$;
- ρ, density;
- σ, defined in equation (13);
- τ, defined in equation (16);
- ψ $= \pi R/L$.

Subscripts

- i, tensor index;
- r, radial component;
- z, axial component;
- 0, inlet condition.

INTRODUCTION

THE problem of flow distribution in gas-cooled nuclear reactor cylindrical channels has been treated in a previous paper [1]. It was shown that if the cooling channels are of the same size the non-uniformity of heat generation in the reactor core distorts the coolant flow in a very unfavourable way, such that hot spots are cooled by a smaller quantity of gas. In order to achieve a more uniform temperature distribution in such reactors, it is therefore necessary to vary the size of the cooling channels or to use suitable throttling devices at certain channels.

In case of the so-called pebble-bed reactors, the coolant flows through the rather irregular space between the pebbles. There are no straight guiding channels for the flow. Since cross-flow is possible in this case, the effect of non-uniform heating will be accentuated and the flow phenomenon is very much more complex than in the case of straight channels.

In this paper the fundamental equations of such a gas flow through a porous mass with non-uniform heat generation will be formulated. One exact solution of these equations for a typical case can be given. In general, however, only numerical solutions using digital computers would be feasible.

THE FUNDAMENTAL EQUATIONS

Consider the steady-state flow of a gas through a pebble-bed reactor. Since the flow space between the pebbles is irregular, it would be impossible to treat the flow field in detail. We shall therefore take the whole pebble-bed as an isotropic porous medium and consider only the mean velocity of gas flow through a unit section perpendicular to the average mass flow. Let u_i be this mean velocity vector.* Then the continuity equation may be written as

$$\operatorname{div}(\rho u_i) = 0$$

$\rho = 1/v$ being the density of the flowing gas. The energy equation, neglecting heat conduction through the mass, is:

$$\rho u_i \, . \operatorname{grad} h = Q$$

where h is the enthalpy per unit mass of the gas and Q is the density of heat generation. Introducing the Nabla operator, the above equations may be written as

$$\nabla_i u_i = \rho u_i \nabla_i v \tag{1}$$

$$\rho u_i \nabla_i h = Q \tag{2}$$

The equation of motion for the flow in a porous medium can be written in the following form in the case of laminar flow [3]:

$$\nabla_i p + a u_i = 0 \tag{3}$$

Namely, the pressure gradient is proportional and parallel to the velocity vector, a being the Darcy coefficient, which is proportional to the viscosity of the gas. For turbulent flow in one-dimensional case, the pressure head drop is related to the velocity head by: [3]†

$$(dp/dx) + F' \rho u^2 = 0 \tag{4}$$

where F' depends on the Reynolds number of flow. A generalization of this expression to a three-dimensional case can be accomplished by assuming that, on account of isotropy, the pressure gradient is in the same direction as the mean flow and that its magnitude depends only on the magnitude of the mass flux density; thus

$$\nabla_i p + F(\rho |u|) u_i = 0 \tag{5}$$

where $F(=F'\rho|u|)$ actually is a function of the Reynolds number, but the dependence on viscosity and on porous structure is ignored in the present study. F may be taken as

$$F = a + b\rho|u| \tag{6}$$

where a and b depend only upon the viscosity of the gas and the structure of the porous mass, The one-dimensional case of (5) reduces to a well-known form [3] of equation (4). Equation (3) is included in (5) if $b = 0$ or at small mass flux.

* No distinction is made between a vector and its components nor between its covariant and contravariant components in case of non-Cartesian co-ordinates [2]. There should be no danger of confusion. The summation convention of tensor notation is understood where it applies.

† For high Mach numbers, a term proportional to $\rho u(du/dx)$ must be included. This term may be neglected in the present application.

One more equation is needed to complete the description of the flow. The coolant will be taken as a polytropic gas (i.e. ideal gas with constant specific heats) or more generally a polytropic vapour [4]. Then the specific enthalpy is given by:

$$h = \lambda p v + \text{const.} \qquad \lambda \equiv k/(k-1) \tag{7}$$

where k is the isentropic exponent.

Equations (1), (2), (5) and (7) are now completely sufficient for the problem. If the boundary conditions and the heat source Q are given, the unknown quantities u_i, p, v, h can be solved. It is evident that an exact solution is impossible in the majority of cases.

In the application to pebble-bed reactors, it can be assumed that the pressure drop through the reactor is only about a few per cent of the absolute pressure [1]. In equation (7) the effect of pressure change on enthalpy can then be neglected and the flow can be considered as practically isobaric with respect to density and temperature changes. Equation (7) can be written as

$$\nabla_i h = \lambda \bar{p} \nabla_i v$$

where $\bar{p}$ is the mean pressure. Substitution into equation (2) results in

$$\rho u_i \nabla_i v = u_i \nabla_i (\log v) = Q/\lambda\bar{p} \tag{8}$$

and, using equation (1),

$$\nabla_i u_i = Q/\lambda\bar{p} \tag{9}$$

We thus obtain the interesting result that the velocity field corresponds to that of an incompressible flow having a mass-source distribution proportional to the actual heat source.

THE CYLINDRICAL REACTOR PROBLEM

We shall now apply the above set of equations to a cylindrical reactor core with radius R and length L, Fig. 1. The cooling gas enters from one end at the cylindrical co-ordinate $z = 0$ and leaves the other end at $z = L$. The heat source density is assumed to be axisymmetrical and is given by [1]:

$$Q(r, z) = \bar{Q} \cdot \frac{\pi}{2} \cdot \frac{s}{2J_1(s)} J_0(ws) \sin \xi \tag{10}$$

where

$$w \equiv r/R, \qquad \xi \equiv \pi z/L \tag{11}$$

and $\bar{Q}$ is the average density of heat source in the entire reactor core. s is the smallest root of the equation

$$J_0(s) = \sigma s J_1(s) \tag{12}$$

and

$$\sigma R \equiv \left[\frac{Q}{-dQ/dr}\right]_{r=R}$$

is the so-called extrapolation length, which depends on the effectiveness of the reflector on the cylindrical surface.

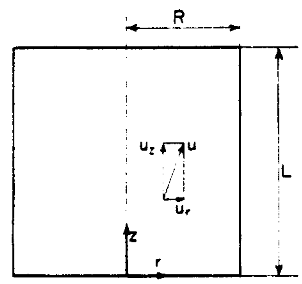

FIG. 1. Cylinder co-ordinates of the reactor core. Gas flows vertically upward.

Using (10), equation (9) in cylindrical co-ordinates becomes

$$\psi \frac{\partial \mu_z}{\partial \xi} + \frac{1}{w} \cdot \frac{\partial (w\mu_r)}{\partial w} = \psi\theta \cdot \frac{s}{4J_1(s)} \cdot J_0(ws) \sin \xi \tag{14}$$

where

$$\left.\begin{aligned} &\mu_z \equiv u_z/\bar{u}_0,\ \mu_r \equiv u_r/\bar{u}_0,\ \psi \equiv \pi R/L \\ &\theta \equiv (\overline{\Delta T})/T_0 = (\pi R^2 L\bar{Q})/(\pi R^2 \bar{u}_0 \lambda \bar{p}) \end{aligned}\right\} \tag{15}$$

u_z, u_r are the axial and radial components of the velocity vector, $\bar{u}_0$ being the mean velocity at entry. θ is the ratio of the mixed mean temperature rise $(\overline{\Delta T})$ to the absolute temperature* T_0 of the gas at entry.

Now define

$$\tau \equiv v/v_0 \tag{16}$$

* This is correct only for polytropic gas. For polytropic vapour, the modification is obvious.

where v_0 is the specific volume at entry. Since the pressure has been assumed constant, τ is also the ratio of absolute temperatures. Substitution of (10) into equation (8) gives

$$\psi\mu_z \cdot \frac{\partial \log \tau}{\partial \xi} + \mu_r \cdot \frac{\partial \log \tau}{\partial w} = = \psi\theta \cdot \frac{s}{4J_1(s)} \cdot J_0(sw) \sin \xi \quad (17)$$

Finally, writing

$$\left.\begin{array}{l} \beta \equiv \pi(p_0 - p)/(\Delta p)_0, (\Delta p)_0 \equiv F(\rho_0\bar{u}_0)\bar{u}_0 L \\ f \equiv F(\rho|u|)/F(\rho_0\bar{u}_0) \end{array}\right\} \quad (18)$$

equation (4) can be separated into its components:

$$\frac{\partial \beta}{\partial \xi} = f\left(\frac{|\mu|}{\tau}\right) \cdot \mu_z, \quad \frac{\partial \beta}{\partial w} = \psi f\left(\frac{|\mu|}{\tau}\right) \mu_r \quad (19)$$

The normalization denominator $(\Delta p)_0$ is evidently the pressure drop which would result if there were no heat generation and the gas conditions at entry could remain unchanged during the through flow.

The four equations (14), (17), (19) now form a simultaneous set for the four unknown quantities $\mu_z, \mu_r, \tau, \beta$. The boundary conditions are

$$\left.\begin{array}{l} \left.\begin{array}{l} \mu_r = 0, \beta = 0 \\ \bar{\mu}_z \equiv \int_0^1 2\, \mu_z w \, dw = 1 \end{array}\right\} \text{for } \xi = 0 \\ \mu_r = \dfrac{\partial \beta}{\partial w} = 0 \text{ for } \xi = \pi, \\ \qquad \text{or } w = 0 \text{ and } w = 1 \end{array}\right\} \quad (20)$$

If equation (6) is used, the function f can be written as

$$f = \frac{1 + \kappa|\mu|/\tau}{1 + \kappa}, \quad \kappa \equiv \frac{b\rho_0\bar{u}_0}{a} \quad (21)$$

κ is a measure of the degree of turbulence and depends upon the Reynolds number at entry.

LAMINAR FLOW

One exact solution is possible for a special form of f, namely if in equation (5) F is constant or in (21) $f = 1$, which is the case if the flow is everywhere laminar and if the change of viscosity with temperature is negligible. Then equation (3) applies and equations (19) become

$$\frac{\partial \beta}{\partial \xi} = \mu_z, \quad \frac{\partial \beta}{\partial w} = \psi\mu_r \quad (22)$$

Taking the divergence of equation (3) or of equations (22) and substituting equation (9) or equation (14), one obtains a Poisson's equation:

$$\nabla_i \nabla_i p + \frac{a}{\lambda \bar{p}} \cdot Q = 0 \quad (23)$$

for the general case and

$$\psi^2 \frac{\partial^2 \beta}{\partial \xi^2} + \frac{1}{w} \cdot \frac{\partial}{\partial w}\left(w \frac{\partial \beta}{\partial w}\right) = \theta \cdot \frac{\psi^2 s}{4J_1(s)} \cdot J_0(sw) \sin \xi \quad (24)$$

for the special case of equation (10).

Equation (24) with the boundary conditions (20) can be readily solved analytically by classical means. The result is

$$\left.\begin{array}{l} \beta(w, \xi) = \left(1 + \dfrac{\theta}{2}\right)\xi - \\ \quad - \dfrac{\theta}{4} \cdot \dfrac{\psi^2 s^2}{\psi^2 + s^2}\left[\dfrac{J_0(ws)}{sJ_1(s)} + \dfrac{I_0(\psi w)}{\psi I_1(\psi)}\right] \sin \xi \\ \mu_z(w, \xi) = \dfrac{\partial \beta}{\partial \xi} = 1 + \dfrac{\theta}{2} - \\ \quad - \dfrac{\theta}{4} \cdot \dfrac{\psi^2 s^2}{\psi^2 + s^2}\left[\dfrac{J_0(ws)}{sJ_1(s)} + \dfrac{I_0(w\psi)}{\psi I_1(\psi)}\right] \cos \xi \\ \mu_r(w, \xi) = \dfrac{1}{\psi} \cdot \dfrac{\partial \beta}{\partial w} = \\ \quad = \dfrac{\theta}{4} \cdot \dfrac{\psi s^2}{\psi^2 + s^2}\left[\dfrac{J_1(ws)}{J_1(s)} - \dfrac{I_1(w\psi)}{I_1(\psi)}\right] \sin \xi \end{array}\right\} \quad (25)$$

J and I are the two Bessel's functions. The correctness of (25) can be easily verified by substituting into equations (20) and (24).

Fig. 2 shows the velocity components μ_z and μ_r for $\sigma = 0$, ($s = 2{\cdot}405$) and $\psi = \pi/2\,(L = R2)$. It may be noted that the velocity field is practically axial if θ is not very much greater than unity.

To determine the temperature field, it will be necessary to solve equation (17) together with (25). The boundary condition is

$$\tau = 1 \text{ for } \xi = 0 \quad (26)$$

This is a differential equation of first order and can be solved numerically or graphically using the well-known isocline method. Owing to the complexity of equation (25) an analytical solution is difficult.

If only the maximum temperature $\tau_m = T_m/T_0$ is needed, then this can be determined analytically. It can be seen that the maximum temperature must occur on the stream line through the cylindrical axis. For this stream line, we have

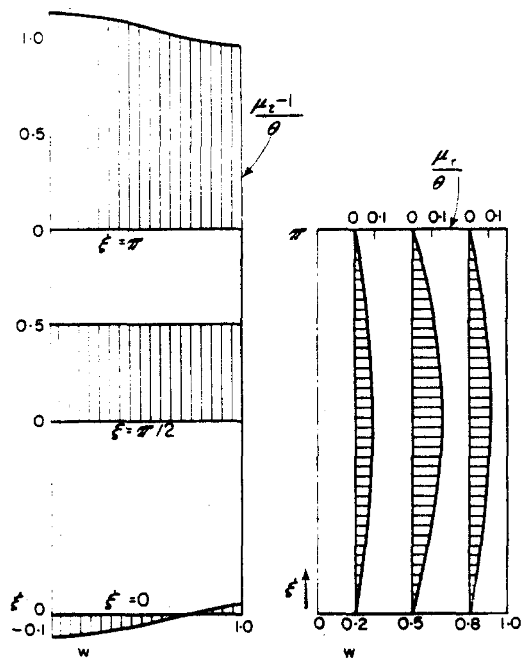

FIG. 2. Coolant velocity field within the reactor core. The plotted values are the axial and radial components of the additional velocity above the adiabatic incompressible values. The additional velocity is proportional to the average heat generation.

evidently $\mu_r = 0$ always due to symmetry. Then equation (17) becomes:

$$\mu_z(0, \xi)\,\frac{d \log \tau}{d\xi} = \frac{\theta}{4} \cdot \frac{s}{J_1(s)} \cdot \sin \xi$$

and substituting μ_z from (25):

$$d \log \tau = \frac{(\theta/4)\,[s/J_1(s)]\,\sin \xi\, d\xi}{(1 + \theta/2) - (\theta/4)\,Z(\psi, s) \cos \xi}$$

$$Z(\psi, s) \equiv \frac{\psi^2 s^2}{(\psi^2 + s^2)}\left[\frac{1}{sJ_1(s)} + \frac{1}{\psi I_1(\psi)}\right] \quad (27)$$

This is readily integrated. The maximum temperature is at the exit point $\xi = \pi$ and its value is

$$\tau_m = \left\{\frac{(1 + \theta/2) + (\theta/4)\,Z(\psi, s)}{(1 + \theta/2) - (\theta/4)\,Z(\psi, s)}\right\}^n$$

$$n \equiv \frac{1 + s^2/\psi^2}{1 + sJ_1(s)/\psi I_1(\psi)} \quad (28)$$

For the data above ($s = 2{\cdot}405$, $\psi = \pi/2$), the evaluation of (28) results in

$$\left.\begin{aligned} \tau_m &= 5{\cdot}15, \text{ for } \theta = 1 \\ \tau_m &= 14{\cdot}7, \text{ for } \theta = 2 \end{aligned}\right\} \quad (29)$$

These values are very much higher than the mixed mean temperature, which is, by definition, given by

$$\bar{\tau} = 1 + \theta \quad (30)$$

The maximum temperature rise can therefore be many times the average temperature rise.

TURBULENT FLOW

The assumption of laminar flow is perhaps non-realistic for the application to pebble-bed reactors. If turbulence is assumed, then the general expression (21) must be used in equation (19), and only numerical solution using digital computer would be feasible. However, some qualitative effects of turbulence may be inferred from the basic equations. Since according to Fig. 2 the magnitude $|\mu|$ of the velocity does not vary very much at sections perpendicular to the axis, the mass flux density in one section will be principally determined by the temperature, i.e. τ in equation (21). Near the axis the heat rate and hence the temperature is high. The frictional force f is smaller according to (21). Equations (19) then show that the velocity there will be higher than in the laminar case. This would mean that the central part gets more cooling gas and the temperature is therefore less acute than in the laminar case. Hence turbulence favours the temperature distribution.

CONCLUSION

It is seen from the calculation that although the velocity field depends mainly upon the average heating rate of the cooling gas and is rather uniform throughout a cross-section of the flow, the temperature field may be quite non-uniform for laminar flow. The maximum value of the temperature rise may be many times the average value. Turbulence in the flow may flatten the temperature peak, but the exact value cannot be determined without a more thorough solution of the given equations.

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